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Grade 12

MATHEMATICS

Paper 1

Topic 2: Sequence & Series

Practice Questions

ALL THE BEST!!

QUESTION 2

- 2.1 Consider the sequence: $\frac{1}{2}; 4; \frac{1}{4}; 7; \frac{1}{8}; 10; \dots$
- 2.1.1 If the pattern continues in the same way, write down the next TWO terms in the sequence. (2)
- 2.1.2 Calculate the sum of the first 50 terms of the sequence. (7)
- 2.2 Consider the sequence: $8; 18; 30; 44; \dots$
- 2.2.1 Write down the next TWO terms of the sequence, if the pattern continues in the same way. (2)
- 2.2.2 Calculate the n^{th} term of the sequence. (6)
- 2.2.3 Which term of the sequence is 330? (4)
- [21]**

QUESTION 3

Given the geometric series: $8x^2 + 4x^3 + 2x^4 + \dots$

- 3.1 Determine the n^{th} term of the series. (1)
- 3.2 For what value(s) of x will the series converge? (3)
- 3.3 Calculate the sum of the series to infinity if $x = \frac{3}{2}$. (3)
- [7]**

QUESTION 2

- 2.1 Tebogo and Matthew's teacher has asked that they use their own rule to construct a sequence of numbers, starting with 5. The sequences that they have constructed are given below.

Matthew's sequence: $5; 9; 13; 17; 21; \dots$

Tebogo's sequence: $5; 125; 3125; 78125; 1953125; \dots$

Write down the n^{th} term (or the rule in terms of n) of:

- 2.1.1 Matthew's sequence (3)
- 2.1.2 Tebogo's sequence (2)
- 2.2 Nomsa generates a sequence which is both arithmetic and geometric. The first term is 1. She claims that there is only one such sequence. Is that correct? Show ALL your workings to justify your answer. (5)
- [10]**

QUESTION 3

Given: $\sum_{t=0}^{99} (3t - 1)$

- 3.1 Write down the first THREE terms of the series. (1)
- 3.2 Calculate the sum of the series. (4)
- [5]

QUESTION 4

The following sequence of numbers forms a quadratic sequence:

$$-3; -2; -3; -6; -11; \dots$$

- 4.1 The first differences of the above sequence also form a sequence. Determine an expression for the general term of the first differences. (3)
- 4.2 Calculate the first difference between the 35th and 36th terms of the quadratic sequence. (2)
- 4.3 Determine an expression for the n^{th} term of the quadratic sequence. (4)
- 4.4 Explain why the sequence of numbers will never contain a positive term. (2)
- [11]

QUESTION 5

Data regarding the growth of a certain tree has shown that the tree grows to a height of 150 cm after one year. The data further reveals that during the next year, the height increases by 18 cm. In each successive year, the height increases by $\frac{8}{9}$ of the previous year's increase in height. The table below is a summary of the growth of the tree up to the end of the fourth year.

	First year	Second year	Third year	Fourth year
Tree height (cm)	150	168	184	$198\frac{2}{9}$
Growth (cm)		18	16	$14\frac{2}{9}$

- 5.1 Determine the increase in the height of the tree during the seventeenth year. (2)
- 5.2 Calculate the height of the tree after 10 years. (3)
- 5.3 Show that the tree will never reach a height of more than 312 cm. (3)
- [8]

QUESTION 2

Consider the following sequence: 399 ; 360 ; 323 ; 288 ; 255 ; 224 ; ...

- 2.1 Determine the n^{th} term T_n in terms of n . (6)
 - 2.2 Determine which term (or terms) has a value of 0. (3)
 - 2.3 Which term in the sequence will have the lowest value? (1)
- [10]

QUESTION 3

- 3.1 Prove that: $a + ar + ar^2 + \dots$ (to n terms) $= \frac{a(r^n - 1)}{r - 1}$, $r \neq 1$ (4)
 - 3.2 Given the geometric series: $3 + 1 + \frac{1}{3} + \dots$
Calculate the sum to infinity. (3)
- [7]

QUESTION 4

Matli's annual salary is R120 000 and his expenses total R90 000. His salary increases by R12 000 each year while his expenses increase by R15 000 each year. Each year he saves the excess of his income.

- 4.1 Represent his total savings as a series. (4)
 - 4.2 If Matli continues to manage his finances this way, after how many years will he have nothing left to save? (3)
 - 4.3 Matli calculates that if his expenses increase by x rand every year (instead of R15 000 each year), he will spend as much as he earns in the 25th year. Determine x . (2)
- [9]

QUESTION 3

The sequence 4 ; 9 ; x ; 37; ... is a quadratic sequence.

- 3.1 Calculate x . (3)
 - 3.2 Hence, or otherwise, determine the n^{th} term of the sequence. (4)
- [7]

QUESTION 2

- 2.1 Evaluate: $\sum_{n=1}^{20} 3^{n-2}$ (4)
- 2.2 The following sequence forms a convergent geometric sequence: $5x ; x^2 ; \frac{x^3}{5} ; \dots$
- 2.2.1 Determine the possible values of x . (3)
- 2.2.2 If $x = 2$, calculate S_{∞} . (2)
- 2.3 The following arithmetic sequence is given: $20 ; 23 ; 26 ; 29 ; \dots ; 101$
- 2.3.1 How many terms are there in this sequence? (2)
- 2.3.2 The even numbers are removed from the sequence.
Calculate the sum of the terms of the remaining sequence. (6)
- [17]**

QUESTION 2

The sequence $3 ; 9 ; 17 ; 27 ; \dots$ is a quadratic sequence.

- 2.1 Write down the next term. (1)
- 2.2 Determine an expression for the n^{th} term of the sequence. (4)
- 2.3 What is the value of the first term of the sequence that is greater than 269? (4)
- [9]**

QUESTION 3

- 3.1 The first two terms of an infinite geometric sequence are 8 and $\frac{8}{\sqrt{2}}$. Prove, without the use of a calculator, that the sum of the series to infinity is $16 + 8\sqrt{2}$. (4)
- 3.2 The following geometric series is given: $x = 5 + 15 + 45 + \dots$ to 20 terms.
- 3.2.1 Write the series in sigma notation. (2)
- 3.2.2 Calculate the value of x . (3)
- [9]**

QUESTION 4

- 4.1 The sum to n terms of a sequence of numbers is given as: $S_n = \frac{n}{2}(5n + 9)$
- 4.1.1 Calculate the sum to 23 terms of the sequence. (2)
- 4.1.2 Hence calculate the 23rd term of the sequence. (3)
- 4.2 The first two terms of a geometric sequence and an arithmetic sequence are the same. The first term is 12. The sum of the first three terms of the geometric sequence is 3 more than the sum of the first three terms of the arithmetic sequence.
- Determine TWO possible values for the common ratio, r , of the geometric sequence. (6)
- [11]**

QUESTION 2

- 2.1 Given the sequence: $4; x; 32$
- Determine the value(s) of x if the sequence is:
- 2.1.1 Arithmetic (2)
- 2.1.2 Geometric (3)
- 2.2 Determine the value of P if $P = \sum_{k=1}^{13} 3^{k-5}$ (4)
- 2.3 Prove that for any arithmetic sequence of which the first term is a and the constant difference is d , the sum to n terms can be expressed as $S_n = \frac{n}{2}(2a + (n-1)d)$. (4)
- [13]**

QUESTION 2

- 2.1 $3x + 1; 2x; 3x - 7$ are the first three terms of an arithmetic sequence. Calculate the value of x . (2)
- 2.2 The first and second terms of an arithmetic sequence are 10 and 6 respectively.
- 2.2.1 Calculate the 11th term of the sequence. (2)
- 2.2.2 The sum of the first n terms of this sequence is -560 . Calculate n . (6)
- [10]**

QUESTION 3

The following sequence is a combination of an arithmetic and a geometric sequence:

$$3 ; 3 ; 9 ; 6 ; 15 ; 12 ; \dots$$

3.1 Write down the next TWO terms. (2)

3.2 Calculate $T_{52} - T_{51}$. (5)

3.3 Prove that ALL the terms of this infinite sequence will be divisible by 3. (2)
[9]

QUESTION 4

A quadratic pattern has a second term equal to 1, a third term equal to -6 and a fifth term equal to -14 .

4.1 Calculate the second difference of this quadratic pattern. (5)

4.2 Hence, or otherwise, calculate the first term of the pattern. (2)
[7]

QUESTION 2

Given the arithmetic series: $-7 - 3 + 1 + \dots + 173$

2.1 How many terms are there in the series? (3)

2.2 Calculate the sum of the series. (3)

2.3 Write the series in sigma notation. (3)
[9]

QUESTION 3

3.1 Consider the geometric sequence: $4 ; -2 ; 1 \dots$

3.1.1 Determine the next term of the sequence. (2)

3.1.2 Determine n if the n^{th} term is $\frac{1}{64}$. (4)

3.1.3 Calculate the sum to infinity of the series $4 - 2 + 1 \dots$ (2)

3.2 If x is a REAL number, show that the following sequence can NOT be geometric:

$$1 ; x + 1 ; x - 3 \dots$$

(4)
[12]

QUESTION 4

An athlete runs along a straight road. His distance d from a fixed point P on the road is measured at different times, n , and has the form $d(n) = an^2 + bn + c$. The distances are recorded in the table below.

Time (in seconds)	1	2	3	4	5	6
Distance (in metres)	17	10	5	2	r	s

- 4.1 Determine the values of r and s . (3)
- 4.2 Determine the values of a , b and c . (4)
- 4.3 How far is the athlete from P when $n = 8$? (2)
- 4.4 Show that the athlete is moving towards P when $n < 5$, and away from P when $n > 5$. (4)
- [13]**

QUESTION 2

- 2.1 Given the geometric series: $256 + p + 64 - 32 + \dots$
- 2.1.1 Determine the value of p . (3)
- 2.1.2 Calculate the sum of the first 8 terms of the series. (3)
- 2.1.3 Why does the sum to infinity for this series exist? (1)
- 2.1.4 Calculate S_{∞} (3)
- 2.2 Consider the arithmetic sequence: $-8 ; -2 ; 4 ; 10 ; \dots$
- 2.2.1 Write down the next term of the sequence. (1)
- 2.2.2 If the n^{th} term of the sequence is 148, determine the value of n . (3)
- 2.2.3 Calculate the smallest value of n for which the sum of the first n terms of the sequence will be greater than 10 140. (5)
- 2.3 Calculate $\sum_{k=1}^{30} (3k + 5)$ (3)
- [22]**

QUESTION 3

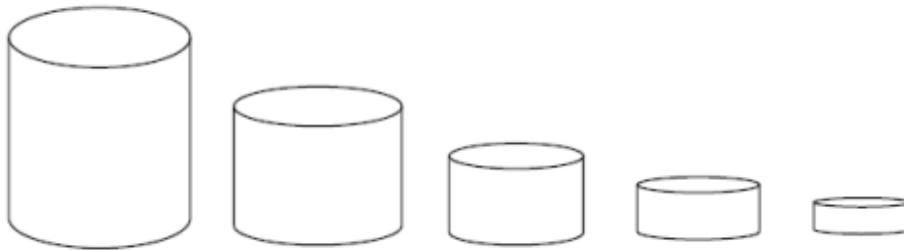
3.1 Given the geometric sequence: $27 ; 9 ; 3 \dots$

3.1.1 Determine a formula for T_n , the n^{th} term of the sequence. (2)

3.1.2 Why does the sum to infinity for this sequence exist? (1)

3.1.3 Determine S_{∞} . (2)

3.2 Twenty water tanks are decreasing in size in such a way that the volume of each tank is $\frac{1}{2}$ the volume of the previous tank. The first tank is empty, but the other 19 tanks are full of water.



Would it be possible for the first water tank to hold all the water from the other 19 tanks? Motivate your answer. (4)

3.3 The n^{th} term of a sequence is given by $T_n = -2(n - 5)^2 + 18$.

3.3.1 Write down the first THREE terms of the sequence. (3)

3.3.2 Which term of the sequence will have the greatest value? (1)

3.3.3 What is the second difference of this quadratic sequence? (2)

3.3.4 Determine ALL values of n for which the terms of the sequence will be less than -110 . (6)

[21]

QUESTION 3

Consider the sequence: $3 ; 9 ; 27 ; \dots$

Jacob says that the fourth term of the sequence is 81.

Vusi disagrees and says that the fourth term of the sequence is 57.

3.1 Explain why Jacob and Vusi could both be correct. (2)

3.2 Jacob and Vusi continue with their number patterns.

Determine a formula for the n^{th} term of:

3.2.1 Jacob's sequence (1)

3.2.2 Vusi's sequence (4)
[7]

QUESTION 2

2.1 Given the geometric sequence: $7 ; x ; 63 ; \dots$

Determine the possible values of x . (3)

2.2 The first term of a geometric sequence is 15. If the second term is 10, calculate:

2.2.1 T_{10} (3)

2.2.2 S_9 (2)

2.3 Given: $0 ; -\frac{1}{2} ; 0 ; \frac{1}{2} ; 0 ; \frac{3}{2} ; 0 ; \frac{5}{2} ; 0 ; \frac{7}{2} ; 0 ; \dots$

Assume that this number pattern continues consistently.

2.3.1 Write down the value of the 191st term of this sequence. (1)

2.3.2 Determine the sum of the first 500 terms of this sequence. (4)

2.4 Given: $\sum_{k=2}^{20} (4x-1)^k$

2.4.1 Calculate the first term of the series $\sum_{k=2}^{20} (4x-1)^k$ if $x = 1$. (2)

2.4.2 For which values of x will $\sum_{k=1}^{\infty} (4x-1)^k$ exist? (3)
[18]

QUESTION 3

3.1 Given the arithmetic sequence: $-3 ; 1 ; 5 ; \dots ; 393$

3.1.1 Determine a formula for the n^{th} term of the sequence. (2)

3.1.2 Write down the 4^{th} , 5^{th} , 6^{th} and 7^{th} terms of the sequence. (2)

3.1.3 Write down the remainders when each of the first seven terms of the sequence is divided by 3. (2)

3.1.4 Calculate the sum of the terms in the arithmetic sequence that are divisible by 3. (5)

3.2 Consider the following pattern of dots:

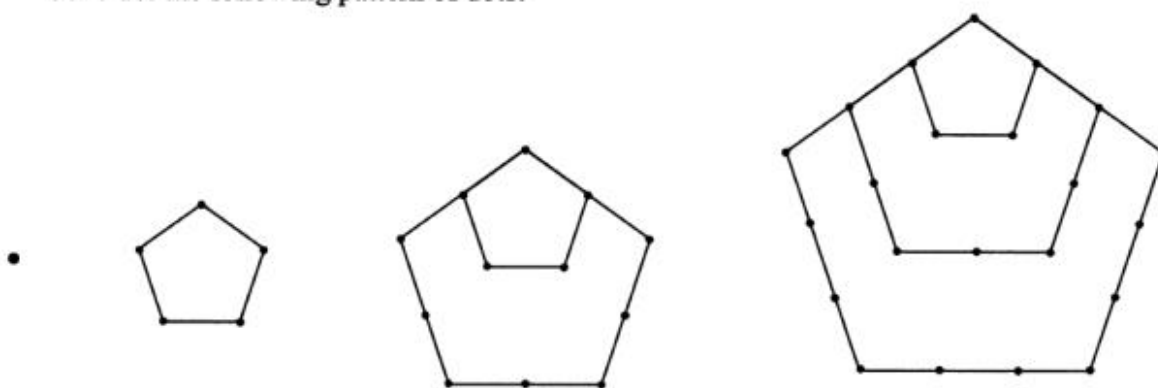


FIGURE 1 FIGURE 2

FIGURE 3

FIGURE 4

If T_n represents the total number of dots in FIGURE n , then $T_1 = 1$ and $T_2 = 5$.
If the pattern continues in the same manner, determine:

3.2.1 T_5 (2)

3.2.2 T_{50} (5)
[18]

QUESTION 2

2.1 A geometric sequence has $T_3 = 20$ and $T_4 = 40$.

Determine:

2.1.1 The common ratio (1)

2.1.2 A formula for T_n (3)

2.2 The following sequence has the property that the sequence of numerators is arithmetic and the sequence of denominators is geometric:

$$\frac{2}{1}; \frac{-1}{5}; \frac{-4}{25}; \dots$$

2.2.1 Write down the FOURTH term of the sequence. (1)

2.2.2 Determine a formula for the n^{th} term. (3)

2.2.3 Determine the 500th term of the sequence. (2)

2.2.4 Which will be the first term of the sequence to have a NUMERATOR which is less than -59 ? (3)
[13]

QUESTION 3

3.1 Given the arithmetic sequence: $w-3$; $2w-4$; $23-w$

3.1.1 Determine the value of w . (2)

3.1.2 Write down the common difference of this sequence. (1)

3.2 The arithmetic sequence 4 ; 10 ; 16 ; ... is the sequence of first differences of a quadratic sequence with a first term equal to 3.

Determine the 50th term of the quadratic sequence. (5)
[8]

QUESTION 4

In a geometric series, the sum of the first n terms is given by $S_n = p \left(1 - \left(\frac{1}{2} \right)^n \right)$ and the sum to infinity of this series is 10.

4.1 Calculate the value of p . (4)

4.2 Calculate the second term of the series. (4)
[8]

QUESTION 2

Given the arithmetic series: $2 + 9 + 16 + \dots$ (to 251 terms).

- 2.1 Write down the fourth term of the series. (1)
 - 2.2 Calculate the 251st term of the series. (3)
 - 2.3 Express the series in sigma notation. (2)
 - 2.4 Calculate the sum of the series. (2)
 - 2.5 How many terms in the series are divisible by 4? (4)
- [12]**

QUESTION 3

- 3.1 Given the quadratic sequence: $-1 ; -7 ; -11 ; p ; \dots$
 - 3.1.1 Write down the value of p . (2)
 - 3.1.2 Determine the n^{th} term of the sequence. (4)
 - 3.1.3 The first difference between two consecutive terms of the sequence is 96. Calculate the values of these two terms. (4)
 - 3.2 The first three terms of a geometric sequence are: $16 ; 4 ; 1$
 - 3.2.1 Calculate the value of the 12th term. (Leave your answer in simplified exponential form.) (3)
 - 3.2.2 Calculate the sum of the first 10 terms of the sequence. (2)
 - 3.3 Determine the value of: $\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{4}\right)\left(1 + \frac{1}{5}\right) \dots$ up to 98 factors. (4)
- [19]**

QUESTION 2

Given the finite arithmetic sequence: $5 ; 1 ; -3 ; \dots ; -83 ; -87$

- 2.1 Write down the fourth term (T_4) of the sequence. (1)
 - 2.2 Calculate the number of terms in the sequence. (3)
 - 2.3 Calculate the sum of all the negative numbers in the sequence. (3)
 - 2.4 Consider the sequence: $5 ; 1 ; -3 ; \dots ; -83 ; -87 ; \dots ; -4\,187$
Determine the number of terms in this sequence that will be exactly divisible by 5. (4)
- [11]**

QUESTION 2

- 2.1 Given the arithmetic series: $18 + 24 + 30 + \dots + 300$
- 2.1.1 Determine the number of terms in this series. (3)
- 2.1.2 Calculate the sum of this series. (2)
- 2.1.3 Calculate the sum of all the whole numbers up to and including 300 that are NOT divisible by 6. (4)
- 2.2 The first three terms of an infinite geometric sequence are 16, 8 and 4 respectively.
- 2.2.1 Determine the n^{th} term of the sequence. (2)
- 2.2.2 Determine all possible values of n for which the sum of the first n terms of this sequence is greater than 31. (3)
- 2.2.3 Calculate the sum to infinity of this sequence. (2)
- [16]**

QUESTION 3

- 3.1 A quadratic number pattern $T_n = an^2 + bn + c$ has a first term equal to 1. The general term of the first differences is given by $4n + 6$.
- 3.1.1 Determine the value of a . (2)
- 3.1.2 Determine the formula for T_n . (4)
- 3.2 Given the series: $(1 \times 2) + (5 \times 6) + (9 \times 10) + (13 \times 14) + \dots + (81 \times 82)$
- Write the series in sigma notation. (It is not necessary to calculate the value of the series.) (4)
- [10]**

QUESTION 2

2.1 Prove that in any arithmetic series in which the first term is a and whose constant difference is d , the sum of the first n terms is $S_n = \frac{n}{2}[2a + (n-1)d]$. (4)

2.2 Calculate the value of $\sum_{k=1}^{50} (100 - 3k)$. (4)

2.3 A quadratic sequence is defined with the following properties:

$$\begin{aligned} T_2 - T_1 &= 7 \\ T_3 - T_2 &= 13 \\ T_4 - T_3 &= 19 \end{aligned}$$

2.3.1 Write down the value of:

(a) $T_5 - T_4$ (1)

(b) $T_{70} - T_{69}$ (3)

2.3.2 Calculate the value of T_{69} if $T_{89} = 23\,594$. (5)
[17]

QUESTION 3

Consider the infinite geometric series: $45 + 40,5 + 36,45 + \dots$

3.1 Calculate the value of the TWELFTH term of the series (correct to TWO decimal places). (3)

3.2 Explain why this series converges. (1)

3.3 Calculate the sum to infinity of the series. (2)

3.4 What is the smallest value of n for which $S_{\infty} - S_n < 1$? (5)
[11]

QUESTION 2

The following geometric sequence is given: $10 ; 5 ; 2,5 ; 1,25 ; \dots$

- 2.1 Calculate the value of the 5th term, T_5 , of this sequence. (2)
 - 2.2 Determine the n^{th} term, T_n , in terms of n . (2)
 - 2.3 Explain why the infinite series $10 + 5 + 2,5 + 1,25 + \dots$ converges. (2)
 - 2.4 Determine $S_{\infty} - S_n$ in the form ab^n , where S_n is the sum of the first n terms of the sequence. (4)
- [10]**

QUESTION 3

Consider the series: $S_n = -3 + 5 + 13 + 21 + \dots$ to n terms.

- 3.1 Determine the general term of the series in the form $T_k = bk + c$. (2)
 - 3.2 Write S_n in sigma notation. (2)
 - 3.3 Show that $S_n = 4n^2 - 7n$. (3)
 - 3.4 Another sequence is defined as:

$Q_1 = -6$
 $Q_2 = -6 - 3$
 $Q_3 = -6 - 3 + 5$
 $Q_4 = -6 - 3 + 5 + 13$
 $Q_5 = -6 - 3 + 5 + 13 + 21$

 - 3.4.1 Write down a numerical expression for Q_6 . (2)
 - 3.4.2 Calculate the value of Q_{129} . (3)
- [12]**

QUESTION 2

- 2.1 Given the following quadratic sequence: $-2 ; 0 ; 3 ; 7 ; \dots$
- 2.1.1 Write down the value of the next term of this sequence. (1)
- 2.1.2 Determine an expression for the n^{th} term of this sequence. (5)
- 2.1.3 Which term of the sequence will be equal to 322? (4)
- 2.2 Consider an arithmetic sequence which has the second term equal to 8 and the fifth term equal to 10.
- 2.2.1 Determine the common difference of this sequence. (3)
- 2.2.2 Write down the sum of the first 50 terms of this sequence, using sigma notation. (2)
- 2.2.3 Determine the sum of the first 50 terms of this sequence. (3)
- [18]**

QUESTION 3

Chris bought a bonsai (miniature tree) at a nursery. When he bought the tree, its height was 130 mm. Thereafter the height of the tree increased, as shown below.

INCREASE IN HEIGHT OF THE TREE PER YEAR		
During the first year	During the second year	During the third year
100 mm	70 mm	49 mm

- 3.1 Chris noted that the sequence of height increases, namely $100 ; 70 ; 49 \dots$, was geometric. During which year will the height of the tree increase by approximately 11,76 mm? (4)
- 3.2 Chris plots a graph to represent the height $h(n)$ of the tree (in mm) n years after he bought it. Determine a formula for $h(n)$. (3)
- 3.3 What height will the tree eventually reach? (3)
- [10]**

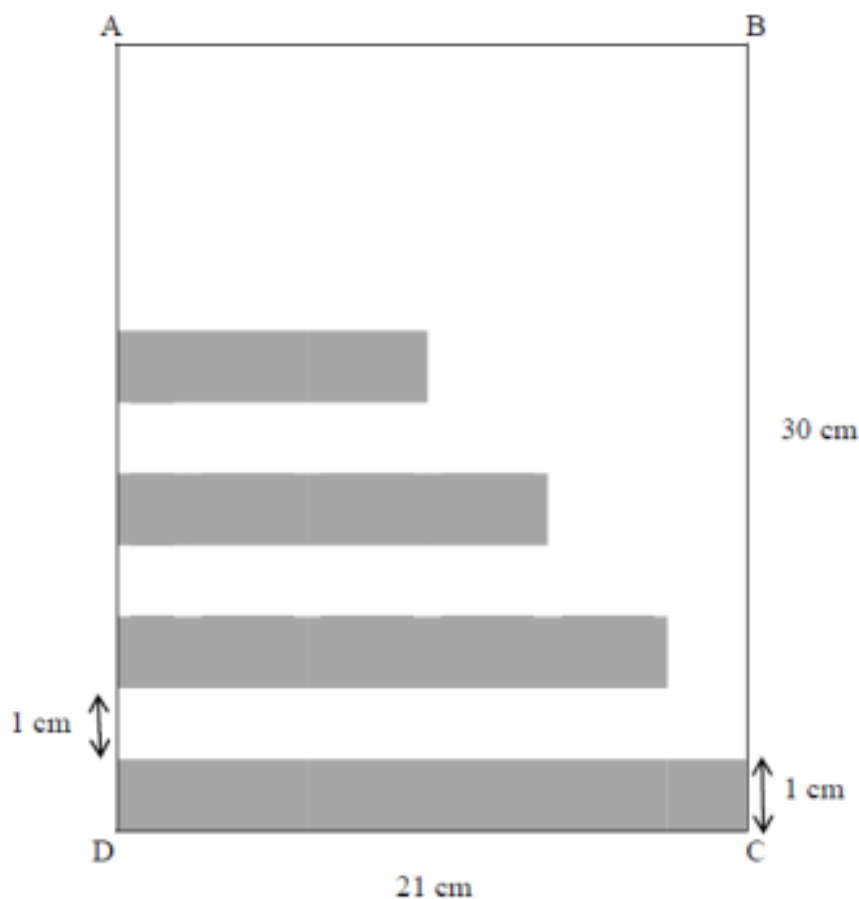
QUESTION 3

3.1 The first four terms of a quadratic number pattern are -1 ; x ; 3 ; $x + 8$

3.1.1 Calculate the value(s) of x . (4)

3.1.2 If $x = 0$, determine the position of the first term in the quadratic number pattern for which the sum of the first n first differences will be greater than 250. (4)

3.2 Rectangles of width 1 cm are drawn from the edge of a sheet of paper that is 30 cm long such that there is a 1 cm gap between one rectangle and the next. The length of the first rectangle is 21 cm and the length of each successive rectangle is 85% of the length of the previous rectangle until there are rectangles drawn along the entire length of AD. Each rectangle is coloured grey.



3.2.1 Calculate the length of the 10th rectangle. (3)

3.2.2 Calculate the percentage of the paper that is coloured grey. (4)
[15]

<<< THE END >>>